

Influence of Magnetic Measurement Modeling on the Solution of Magnetostatic Inverse Problems Applied to Current Distribution Reconstruction in Switching Air Arcs

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This paper presents a systematical analysis of the influence of magnetic measurement modeling on the regularized solution of magnetostatic inverse problems to reconstruct a current distribution. The focus is on the modeling process of a planar sensor array used to perform the spatially resolved magnetic measurements considering the active area of sensors. The results show inversion error produced by the modeling process becomes more significant when the signal-to-noise ratio is large.

Index Terms—Magnetostatic, inverse problem, measurement modeling, magnetic sensor, active area.

I. INTRODUCTION

MAGNETOSTATIC inverse problems are concerned with the reconstruction of static or slowly time-varying current distributions, in regions not accessible to direct measurement, from measurements of their magnetic fields. A typical example is the identification of current profiles in arc plasmas inside circuit breakers [1, 2]. In previous simulation works, spatially resolved measurements of magnetic field are assumed to be performed by some ideal magnetic sensors locating on the proximity of test object under interest, i.e., the geometry size or active area of the sensor are not considered [3]. But, on-chip magnetic sensors generally used in practical applications measure the total magnetic flux through their active area or the average magnetic flux density over their active area [4]. Then the assumption of ideal sensor model may lead to large errors in the modeling process and finally in the inverse solutions.

Therefore, the aim of this paper is to systematically analyze the influence of modeling process of magnetic measurements, considering the geometry size or active area of sensors, on the solution of the magnetostatic inverse problem for the reconstruction of current distribution in arc plasmas.

II. METHODS

A. Forward and Inverse Problem

The magnetostatic inverse problem of reconstructing the discretized currents vector \mathbf{J} distributed over a simplified arc chamber based on the vector of magnetic measurement values \mathbf{B} with the ideal sensor model corresponding to

$$\mathbf{B} = \mathbf{W} \cdot \mathbf{J} + \boldsymbol{\delta} \quad (1)$$

in which $\boldsymbol{\delta}$ represents the measurement perturbation and \mathbf{W} the lead field matrix which is a finite dimensional discrete analogue of Biot-Savart operator incorporating information on the sensor array, the source space grid, and the forward model. Using the finite integration technique (FIT) [5] for the discretization by assuming a network of wires l_j with current J_j flowing through, \mathbf{W} can be computed for i -th sensor and j -th element of source domain as

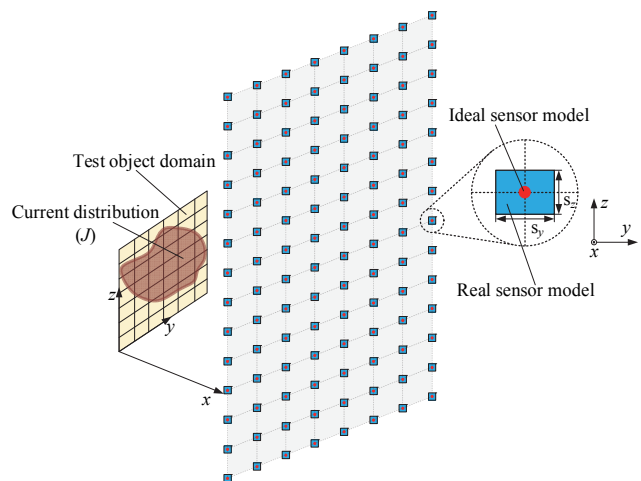


Fig. 1. Schematic diagram of the geometry model and the planar sensor layout. The test object domain is 40 mm and 30 mm in y and z direction, respectively, with the discretization levels of $n_y = 200$ and $n_z = 100$. The measurement system is a 16×16 planar sensor array.

$$W_{ij} = \omega_i \mathbf{d}_i \mathbf{L}_{ij},$$

$$\mathbf{L}_{ij} = \frac{\mu_0}{4\pi} \int_{l_j} \frac{\mathbf{e}_j \times (x_i - x_s, y_i - y_s, z_i - z_s)^T}{\left((x_i - x_s)^2 + (y_i - y_s)^2 + (z_i - z_s)^2 \right)^{3/2}} dl_j, \quad (2)$$

where \mathbf{d}_i is the i -th sensor direction, ω_i the sensor weight, and $\mathbf{r}_i := (x_i, y_i, z_i)$ the position of the sensor, \mathbf{e}_j the direction of j -th element l_j .

For the real sensor model, the magnetic sensors is assumed to measure the average magnetic flux density \mathbf{B} over its active area. We confine a 2D rectangular geometry as the real sensor model, and each sensor is located where its geometry center is assumed to be coincide with the ideal sensor point, i.e., the center of i -th real sensor model is coincide with $\mathbf{r}_i = (x_i, y_i, z_i)$ as shown in Fig.1. The default sensor direction is set to be the same as the ideal sensor model. Then the problem using real sensor model can be defined as

$$\mathbf{B}_\phi = \boldsymbol{\Psi} \mathbf{J} + \boldsymbol{\delta}, \quad \text{with } \Psi_{ij} = \int_{S_i} \omega_i \mathbf{d}_i \mathbf{L}_{ij} ds_i / |S_i| \quad (3)$$

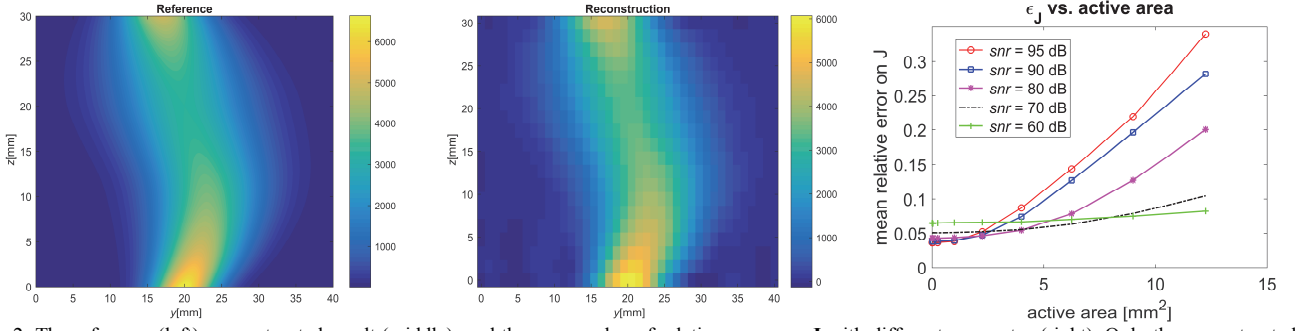


Fig. 2. The reference (left), reconstructed result (middle), and the mean value of relative errors on \mathbf{J} with different parameters(right). Only the reconstructed \mathbf{J}_α with $l_s = 2.0$ mm and $snr = 80$ dB are shown to limit the number of presented graphs.

in which \mathbf{B}_ϕ is the vector of measurements values of average magnetic flux density, s_i the active area of i -th sensor.

We investigate in what range of the active area of sensors is that makes no difference on the solution when the sensors are assumed to be an ideal point sensor model. Thus, we solve (1) but with \mathbf{B} replaced by \mathbf{B}_ϕ computed from (3).

B. Regularized Solution

The inverse problem of (1) is known to be ill-posed with an extremely large condition number of \mathbf{W} . So the Tikhonov regularization technique is used to yield a least squares solution defined as

$$\mathbf{J}_\alpha = \arg \min_{\mathbf{J}} \left\{ \left\| \mathbf{W}\mathbf{J} - \mathbf{B}_\phi \right\|_2^2 + \alpha^2 \left\| \mathbf{J} \right\|_2^2 \right\} \quad (4)$$

To evaluate the inverse solution \mathbf{J}_α , we compute the relative error for the reconstructed currents from

$$\varepsilon_J = \left\| \mathbf{J}_\alpha - \mathbf{J} \right\|_2 / \left\| \mathbf{J} \right\|_2 \quad (5)$$

III. NUMERICAL RESULTS AND DISCUSSION

A 2D planar spatial source grids are considered with an exponential profile of current distribution as the reference as shown in Fig.1. A Gaussian noise of δ with a signal-to-noise ratio (SNR) $snr \in \{95, 90, 80, 70, 60\}$ dB is added to measurement values. The impact of active area on ε_J is analyzed with the active area $s = l_s^2$ of $l_s \in \{0.05, 0.1, 0.2, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5\}$ mm. The relative errors are applied in 100 repeated runs each, and we compute the mean values of ε_J for each case. Numerical results are shown in Fig.2.

In the explored range of l_y (0.05 mm ~3.5 mm), the relative errors are observed to increase quickly with the increment of s when snr is large. For the solution of the inverse problem, the modeling process with real sensor models is of much complexity. So we are interested in the case when the active area of sensors can be ignored or assumed to be an ideal point sensor, i.e., there is no difference in ε_J between the ideal sensor model and the real sensor model. Qualitatively, if the magnitude of the gradient of magnetic field at the measurement plane is relatively small, sensors with large active areas would not lead to a much larger relative error when we perform the modeling process with the simple ideal sensor model.

Due to the ill-posedness of \mathbf{W} , the Tikhonov regularization is to approximate the unbounded operator pseudoinverse \mathbf{W}^{-1}

by a bounded operator \mathbf{R}_α with the regularization parameter $\alpha \geq 0$. From (4), the regularization strategy is defined by

$$\mathbf{J}_\alpha = \mathbf{R}_\alpha \mathbf{B}_\phi, \quad (6)$$

$$\mathbf{R}_\alpha = \left(\alpha^2 \mathbf{I} + \mathbf{W}^* \mathbf{W} \right)^{-1} \mathbf{W}^* \text{ with } \left\| \mathbf{R}_\alpha \right\|_2 \leq \frac{1}{2\sqrt{\alpha}}. \quad (7)$$

The error for the reconstruction can be estimated by

$$\begin{aligned} \left\| \mathbf{J}_\alpha - \mathbf{J} \right\|_2 &= \left\| \mathbf{R}_\alpha \mathbf{B}_\phi - \mathbf{R}_\alpha \mathbf{W}\mathbf{J} + \mathbf{R}_\alpha \mathbf{W}\mathbf{J} - \mathbf{J} \right\|_2 \\ &\leq \left\| \mathbf{R}_\alpha \mathbf{B}_\phi - \mathbf{R}_\alpha \mathbf{B} \right\|_2 + \left\| \mathbf{R}_\alpha \mathbf{W}\mathbf{J} - \mathbf{J} \right\|_2 \quad (8) \\ &\leq \frac{1}{2\sqrt{\alpha}} \left\| \mathbf{B}_\phi - \mathbf{B} \right\|_2 + \left\| \mathbf{R}_\alpha \mathbf{W}\mathbf{J} - \mathbf{J} \right\|_2. \end{aligned}$$

The first term on the right-hand side of (8), called perturbation error, describes the error coming from the data noise and the modeling process of neglecting the active area (i.e., replacing \mathbf{B} by \mathbf{B}_ϕ). The second term, called regularization error, denotes the approximation error (replacing \mathbf{W}^{-1} by \mathbf{R}_α) which purely depends on the regularization strategy \mathbf{R}_α . The larger is the measurement perturbation (with smaller snr), the inversion error produced by the modeling process becomes more insignificant. More care should be taken when modeling a measurement system with high snr .

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